A COMBINED GAUSS-NEWTON AND QUASI-NEWTON INVERSION METHOD FOR THE INTERPRETATION OF APPARENT RESISTIVITY PSEUDOSECTIONS

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INTRODUCTION

Two-dimensional (2D) electrical imaging surveys (Griffiths and Barker 1993) are now widely used to map areas with moderately complex geology where conventional resistivity sounding surveys do not give sufficiently accurate results. In more complex areas three-dimensional (3D) surveys (Ellis and Oldenburg 1994, Loke and Barker 1996b), which gives even more accurate results but at a greater cost, have also been used.

The smoothness-constrained Gauss-Newton least-squares method has been successfully used for 2D and 3D inversion of apparent resistivity data (deGroot-Hedlin and Constable 1990, Sasaki 1994). One disadvantage of this technique is the large computing time required for the calculation of the partial derivatives and to solve the least-squares equation. Loke and Barker (1996a,b) used a quasi-Newton method to estimate the partial derivatives to reduce the computing time. However, since the Gauss-Newton method uses the exact partial derivatives, it should produce more accurate results. In this paper, a method which combines the accuracy of the Gauss-Newton method with the speed of the quasi-Newton method is examined.

THE LEAST-SQUARES METHOD

The least-squares formulation used in this research, which constrains the smoothness of the model parameters to a constant value, is given by the following equation.

$$(\mathbf{J}_i^T\mathbf{J}_i + \lambda_i \mathbf{C}^T\mathbf{C})\mathbf{p}_i = \mathbf{J}_i^T\mathbf{g}_i - \lambda_i \mathbf{C}^T\mathbf{C}\mathbf{r}_{i-1}, \qquad (1)$$

where $\mathbf{J}_i$ is the Jacobian matrix of partial derivatives, $\mathbf{C}$ is the flatness filter matrix, $\mathbf{g}_i$ is a vector which contains the differences between the logarithms of the measured and calculated apparent resistivity values, λ_i is the damping factor, $\mathbf{p}_i$ is the perturbation vector to the model parameters for the i th iteration, and $\mathbf{r}_{i-1}$ is the model parameters vector for the previous iteration. In the Gauss-Newton least-squares method, the Jacobian matrix is recalculated after each iteration by the finite-difference or finite-element method. To reduce the computing time, Loke and Barker (1996a,b) used a quasi-Newton updating method to estimate the Jacobian matrix after each iteration. A homogeneous earth model, for which the Jacobian matrix values can be calculated analytically, is used as the starting model. After each iteration, the Jacobian matrix is estimated by using the following updating equation

$$\mathbf{B}_{i+1} = \mathbf{B}_i + \mathbf{u}_i\,\mathbf{p}_i^T, \qquad (2)$$

where $\quad \mathbf{u}_i = (\Delta\mathbf{y}_i - \mathbf{B}_i \mathbf{p}_i) / \mathbf{p}_i^T \mathbf{p}_i \,, \qquad \Delta\mathbf{y}_i = \mathbf{y}_{i+1} - \mathbf{y}_i,$

and $\mathbf{B}_{i+1}$ is the estimated Jacobian matrix for the $(i+1)$th iteration, $\mathbf{y}_i$ is the model response and $\Delta\mathbf{y}_i$ is the change in the model response for the ith iteration (Broyden 1965). The time taken to solve the least-squares equation (1) is also reduced by using a similar updating method. The convergence rate of the quasi-Newton method is slower than the Gauss-Newton method but the time taken per iteration can be much less. In areas with subsurface resistivity contrasts of less than 10, there were no significant differences in the results obtained by the two methods while the computer time taken by the quasi-Newton method can be an order of magnitude smaller (Loke and Barker 1996a). In this paper, a comparison of the results obtained with both methods for cases with a much larger resistivity contrast is made.

RESULTS

Figure 1 shows a test model consisting of a rectangular prism with a resistivity of 500 ohm.m embedded in a medium with a resistivity of 10 ohm.m. The apparent resistivity values measured with a multi-electrode system with 51 electrodes using the Wenner array are calculated using a finite-difference program. All the possible 408 apparent resistivity values for electrode spacings of 1 to 16 metres is used as the input data set (Figure 2a). Gaussian random noise of 1% was added to the apparent resistivity values.

The inversion model which consists of 400 rectangular blocks is also shown in Figure 1. Figure 2 shows the models obtained with the Gauss-Newton and quasi-Newton methods. The change in the RMS error with iteration number for the Gauss-Newton and quasi-Newton inversion methods are shown in Figure 3. The starting homogeneous earth model gives an apparent resistivity RMS error of 56.5 % which is progressively reduced after each iteration. The largest reductions in the RMS error occur in the first few iterations. From the 3rd iteration onwards, the RMS error achieved by the Gauss-Newton method is significantly lower than that obtained with the quasi-Newton method. The Gauss-Newton method converges in 6 iterations with an RMS error of about 1.2%. In comparison, the RMS error for the quasi-Newton method at the 6th iteration is about 2.8%, after which it slowly decreases to an asymptotic value of about 1.5% after 9 iterations.

Figure 4 shows the change in the resistivity value of a selected model block (shaded in Figure 1) with iteration number. The change in the partial derivative value for one of the apparent resistivity values is also shown. The block resistivity and the partial derivative values show the largest changes in the first 2 or 3 iterations. Figure 3 also shows the change in the RMS error when partial derivatives are recalculated by the finite-difference subroutine for the 1st iteration only, after which it is estimated by the quasi-Newton method. The error curve for this hybrid method lies in between the error curves for the Gauss-Newton and quasi-Newton methods (Figure 3). The convergence rate is further improved by recalculating the partial derivatives for the first 2 iterations. When the partial derivatives are recalculated for the first 3 iterations, there is no significant difference with the results obtained with the Gauss-Newton method. Since the largest changes occur in the first 2 or 3 iterations, the error in the partial derivative values estimated by the quasi-Newton method in the later iterations has a smaller effect on the results. Similar results were obtained for other 2D test models and field data sets, as well as in the 3D inversion of synthetic and field data sets.

CONCLUSIONS

Recalculating the partial derivatives for the first 2 or 3 iterations represents a good compromise between reducing the computing time and obtaining sufficiently accurate results. The computer time is reduced by about half, which is particularly important in 3D resistivity inversion which can involve more than 10000 datum and very large finite-difference grids.

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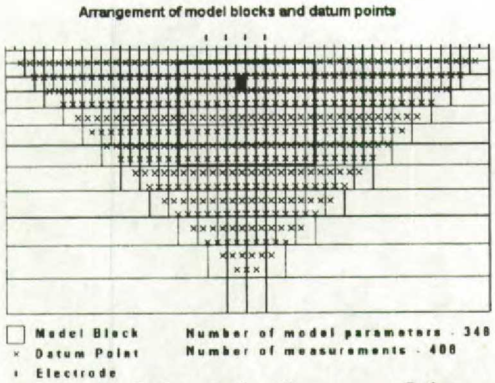

Figure 1. Schematic diagram of the rectangular prism test model together with the arrangement of the blocks used by the inversion program.

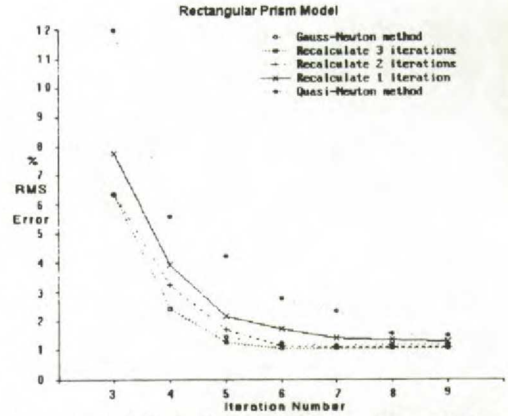

Figure 3. The change of the apparent resistivity RMS error with iteration number for the different inversion methods.

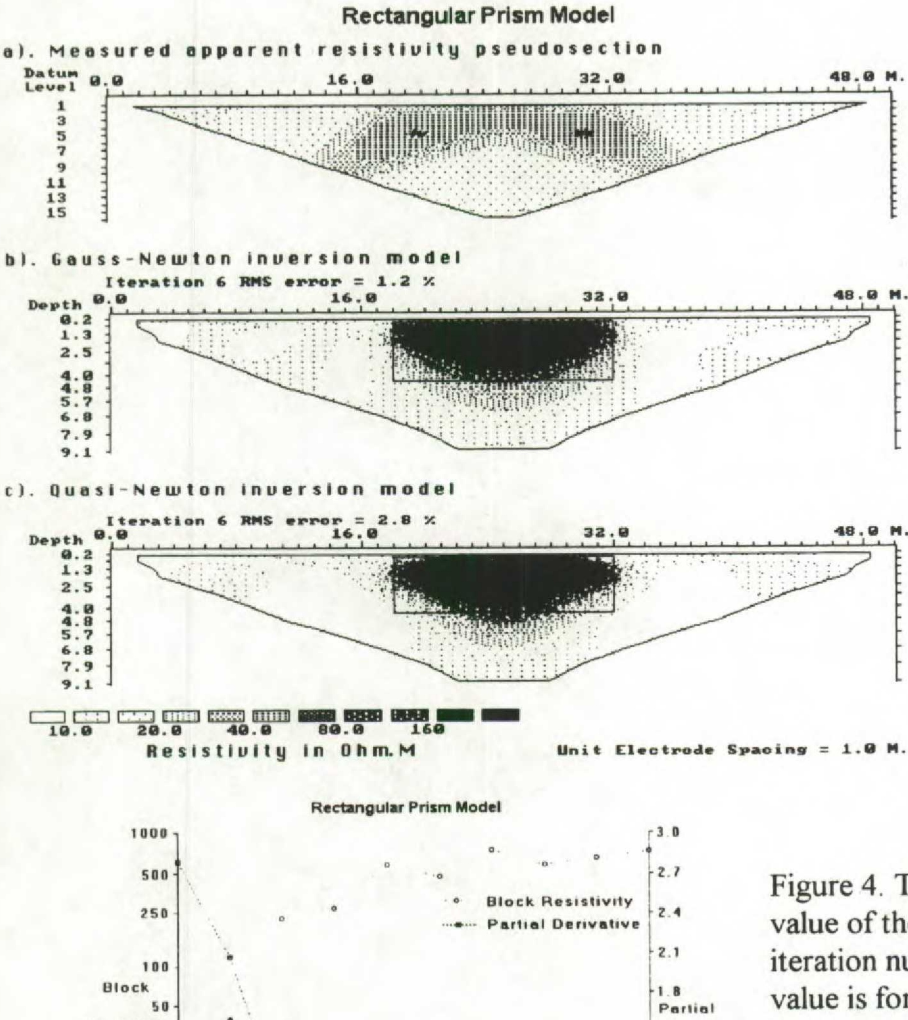

Figure 2.
(a) Measured apparent resistivity pseudosection
(b) Model obtained with Gauss-Newton method.
(c) Model obtained with quasi-Newton method.

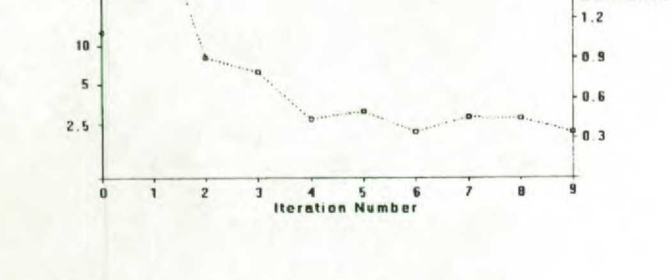

Figure 4. The change in the resistivity value of the shaded block in Figure 1 with iteration number. The partial derivative value is for the measurement made with the four electrodes marked in Figure 1.