# The location of infinite electrodes in pole-pole electrical surveys and the resulting error for 2D electrical imaging. A practical point of view. 

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## INTRODUCTION.

The improvement of multi-electrode arrays for resistivity measurements has led to an important development of electrical imaging for subsurface surveys. These arrays produce wide apparent resistivity pseudosections, presenting a large set of close data. It allows precise direct or inverse interpretation of 2D ground resistivity at shallow depth.

The pole-pole array (P2) is frequently used. Several reasons explain this choice : (i) the depth of penetration of this array is greater than any other, (ii) this array minimizes edge effects and thus provides the clearest anomaly shapes in pseudosections, (iii) the large MN distance provides also high signal/noise ratio, and (iv) the result for any other array may be derived from pole-pole array results. This array corresponds to a quadripole A B (current electrodes) M N (potential electrodes) where B and N are placed "at infinity" so that the presence of these electrodes may be ignored.

Literature states that electrodes located «at infinity» should be at least 10 to 20 times further away than AM distance (Keller and Frischknecht 1966). In facts, for many field surveys, the electrodes located «at infinity» hardly reach more than 5 to 10 times the greatest AM distance. These finite arrays, more or less approaching P2, may be called pseudo polepole arrays (P3). The electrical image obtained using P3 may be strongly distorted with respect to the one corresponding to P 2 . Consequently, the use with P 3 data set of 2 D direct or inverse modelling software designed for P2 data could lead to erroneous interpretation.

This paper, based on computer numeric simulation, which allows to calculate P2 and P3 values, aims at quantifying these distortions and at proposing some solutions to obtain with P3 the field data set closest to the one which should be obtained with P2.

## METHODS

Our simulation used multi-electrode arrays with 23 electrodes. A and $20 \mathrm{M}_{\mathrm{i}}$ electrodes were fixed in line every 2 m while B and N were located further away in different positions. These arrays provide a 20 measurement points shallow depth sounding. Measurements points are conventionally drawn at $\mathrm{AM}_{\mathrm{i}}$ center and at a depth corresponding to $\mathrm{AM}_{\mathrm{i}} / 2$ (Fig.1). A pseudosection (or 2D electrical image) corresponds to a large number of such soundings


Figure 1 acquired this a step corresponding to the smallest $\mathrm{AM}_{\mathrm{i}}$ spacing.

The 20 apparent resistivities were calculated using the general equation :

$$
\rho=K . \Delta V / I(1) .
$$

$\Delta V / I$ values were calculated using the potential values computed at $M_{i}$ and $N$ locations for a unit current flowing through $\mathrm{A}(+)$ and $\mathrm{B}(-)$ into a ground made up of 3 layers with varying thickness and resistivity. The general moment method developed for electromagnetic 3D modeling was used (Tabbagh 1985) but restricted in our simulation for contrast in electrical conductivity only.

K , the geometric coefficient of each quadripole, was calculated as following :

$$
\mathrm{K}=2 \pi /(1 / \mathrm{AM}-1 / \mathrm{BM}-1 / \mathrm{AN}+1 / \mathrm{BN})(2)
$$

For P 2 , where $1 / \mathrm{BM}, 1 / \mathrm{AN}$ and $1 / \mathrm{BN}$ are nil, (2) may be simplified to :

$$
\mathrm{K}=\mathrm{K} \alpha=2 \pi \mathrm{AM}(3)
$$

For P3 the approximation $\mathrm{K}=\mathrm{K} \alpha$ may be strongly erroneous. In these cases, the use of (2) to calculate $\rho$ is the only way which physically makes sense.

## RESULTS AND DISCUSSION.

The apparent resistivity measured with P2 is generally different from the one measured with P3. The error made when assuming that those two values are equal is :

$$
E(\rho)=E(K)+E(\Delta V / I)(4
$$

The error for the geometric coefficient is :

$$
E(K)=(K-K \alpha) / K(5)
$$

The calculation using AM, AN, BM and BN analytical expressions with the conventions presented in fig.2, gives :


Figure 2

$$
E(K)=\frac{1}{Q} \cdot\left(\frac{2}{\sqrt{1+\frac{2 \cdot \cos v}{}+\frac{1}{Q^{2}}}}+\frac{2}{\sqrt{1-\frac{2 \cdot \cos \beta}{} \mathrm{Q}+\mathrm{Q}^{2}}}-\frac{1}{\sin \left|\begin{array}{c}
v-\beta  \tag{6}\\
2
\end{array}\right|}\right)
$$

For $x$ close to zero, the approximation $(1+x)^{a} \approx 1+a . x$ may be used. It comes :

$$
\begin{equation*}
E(K) \approx \frac{1}{Q} \cdot\left(4-\frac{1}{\sin \left|\frac{v-\beta \mid}{2}\right|}+\frac{2}{Q} \cdot(\cos \beta-\cos v)\right) \tag{7}
\end{equation*}
$$

At the first order or for $\beta=-v$, it appears that $E(K)$ only depends on $Q$ and $\theta=|v-\beta|$. Particularly, it appears that it exists a P3 which have exactly the same geometric coefficient than P2 with any Q. $\theta=\theta_{0}=28.96^{\circ}$, is actually a sufficient condition to cancel $E(K)$.

The computation of $\mathrm{E}(\rho)$ was conducted for different ground models, with fixed Q and varying $\theta$. The results for two ground models are presented in fig.3. The most striking point is that for the same $\mathrm{E}(\mathrm{K}), \mathrm{E}(\rho)$ may vary a lot depending on ground properties. This shows that, at the contrary of $\mathrm{E}(\mathrm{K})$ which only depends on array geometry, $\mathrm{E}(\rho)$ cannot a priori be estimated without assuming an hypothesis for ground model. This is not reliable as the ground model is the very purpose of interpretation. The curves present, however, some interesting similarities: $\rho$ is generally underestimated when P 3 is used instead of P 2 . $\mathrm{E}(\rho)$ is very high when $\theta$ is small. The curves seems to reach a minimum value for the same angle $\theta=\theta_{0} / 2$. They also reach a relative maximum for $\theta=180^{\circ}$. This last fact shows that the array generally used for pole-pole surveys is not at all the most suitable.


Figure 3 : Errors for K and $\rho$ versus $\theta$ (index $\alpha$ corresponds to true pole-pole array)

Considering the 20 measurement points, with N and B fixed so that $\mathrm{E}\left(\mathrm{K}_{\mathrm{i}}\right)=0$ for the greatest $\mathrm{AM}_{\mathrm{i}}$, it appears that $\mathrm{E}(\rho)$ standard deviation vary a lot according to $\theta$ (Fig.4). The weakest standard deviation is not observed for $\theta_{0} / 2$, minimising $\mathrm{E}\left(\rho_{\mathrm{i}}\right)$ for the greatest $\mathrm{AM}_{\mathrm{i}}$, but for $\theta_{0}$, cancelling $\mathrm{E}\left(\mathrm{K}_{\mathrm{i}}\right)$ for the greatest $\mathrm{AM}_{\mathrm{i}}$. The use of this particular P3 thus provides a shifted P2 data set but not a distorted one. One should however note that the shift cannot a priori be estimated since it depends on ground properties.


Figure 4 : mean error for $\rho$ versus $\theta$

## CONCLUSION

In electrical imaging survey, the available cable length generally impose a poor approximation for the frequently used pole-pole array. Our study shows that in most of the cases it provides an erroneous image with an underestimation of apparent resistivities increasing with depth. But it also appears that a particular finite multi-electrode array may provide results similar to the ones of the impracticable pole-pole array. The array is made of $\mathrm{n}+3$ electrodes : $\mathrm{A}, \mathrm{M}, \ldots, \mathrm{M}_{\mathrm{n}}$ are regularly spaced in line ; B and N are placed on both sides of $A M_{i}$ line so that $B \bar{O} N \approx 30^{\circ}$ with O at the middle of $\mathrm{AM}_{n}$. The electrical image obtained with this unusual array is actually shifted but not distorted with respect to pole-pole image. The interpretation of this data set with direct or inverse software designed for pole-pole data will thus provide an accurate interpretation of the ground geometry. The shift cannot a priori be estimated because it depends on the ground geoelectrical structure. The actual resistivities of the different bodies may thus be determined with complementary logging methods.

## REFERENCES

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