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Attenuating noise of multicomponent seismic data using F-X domain SVD

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SUMMARY

This paper presents a method of random noise attenuation of multicomponent seismic data. A spectral covariance matrix of seismic wavefield data in F-x domain is formed and in order to avoid dealing with very large matrixes, the reduced dimensional spectral covariance matrix is estimated by means of singular value decomposition (SVD). By finding the highest eigenvalue of the reduced dimensional covariance matrix we are able to separate the desired seismic waves from the noise. The results have shown that the proposed algorithm outperforms the conventional separation technique in terms of accuracy and complexity.

Introduction

In seismic exploration, recorded geophysical data is often contaminated by random noise. The conventional basic Singular Value Decomposition (SVD) has been dramatically used in time domain to attenuate noise on multicomponent seismic wavefield data. Similar filtering in frequency domain is for mono frequency signal called spectral-matrix filtering (SMF) and for band limited signal is called wide band spectral-matrix filtering (WBSMF) **(Paulus et al., 2005)**. For multi component multi band data they are computationally expensive to diagonalize the whole spectral matrix. The proposed method in this paper is based on the MC-WBSMF algorithm **(Paulus et al., 2005; Paulus and Mars, 2006)**. It reduces the dimension of the multicomponent spectral covariance matrix and estimates it by means of the SVD technique in frequency domain **(Aws Al-Qaisi, 2009)**.

Method

In order to produce a three component synthetic seismic data, various frequencies of three different components has been combined **(figure 1)** and we use a synthetic signal composed of three waves with different reciprocal ellipticities. One of them is linearly polarized; meanwhile the other two have elliptical polarization.

By applying the Fourier transform, a multicomponent seismic model can be represented in F-x domain as: (k_d : number of components, k_x : number of sensors, k_f : number of frequency bins)

$$Y_f = FT\{Y_t\} \in d^G \quad , \quad G = k_d k_x k_f \tag{1}$$

The information can be rearranged into one long vector as follow:

$$y_f = \left[h(f_1)^T \ldots h(f_{k_f})^T, v(f_1)^T \ldots v(f_{k_f})^T, z(f_1)^T \ldots z(f_{k_f})^T \right]^T \tag{2}$$

Interactions between different components of directional sensors in all frequencies can be stated in a multi-component covariance spectral matrix defined by:

$$E = E(yy^H) \tag{3}$$

Spatial and frequency smoothing operator can be applied to perform an estimation of noninvertible unity rank spectral covariance matrix **(figure 3,4)(C. Paulus, 2005)**.

$$\hat{E} = \tfrac{1}{N} \sum_{n_s=1}^{2N_s+1} \sum_{n_f=1}^{2N_f+1} y_{n_s,n_f} \, y^H_{n_s,n_f} \quad , \quad N = (2N_s+1)(2N_f+1) \tag{4}$$

Where N_s is spatial smoothing order and N_f is frequency smoothing order.

It is computationally expensive to diagonalize the whole estimated spectral covariance matrix $\hat{E}$ of dimension (G×G). For that reason, a new matrix R of size (G× N) that contains concatenated long-vectors resultant from the spatial and frequency smoothing can be proposed as: **(Aws Al-Qaisi, 2009)**

$$R = \left[y_{1,1} \cdots y_{2N_s+1,1} \cdots y_{1,2N_f+1} \cdots y_{2N_s+1,2N_f+1} \right]_{G\times N} \tag{5}$$

Matrix R is used to obtain $\hat{E}$ as below:

$$\hat{E} = \tfrac{1}{N} R.R^H \tag{6}$$

Reduced dimensional spectral covariance matrix $\breve{E}$ of dimension $N \times N$ is introduced with the same amount of eigenvalue of matrix $\hat{E}$ **(Aws Al-Qaisi, 2009)**.

$$\breve{E} = \tfrac{1}{N} R^H . R \tag{7}$$

Substituting of the SVD of matrix R=USV^H one gets:

$$\breve{E} = \tfrac{1}{N} R^H . R = \tfrac{1}{N} VS^H U^H USV^H = \tfrac{1}{N} VS^2 V^H = \tfrac{1}{N} V \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_N^2 \end{bmatrix} V^H \tag{8}$$

The eigenvectors of the reduced dimensional estimated spectral covariance matrix can now be stated as:

$$U = RV^H \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_N \end{bmatrix} \tag{9}$$

As a result, the filtering step corresponds to an orthogonal projection of a long vector y_f onto the eigenvectors in U that have the highest eigenvalues **(figure5)**.

$$y_{final} = \sum_{i=1}^{p} \langle y_f, u_i \rangle u_i = \sum_{i=1}^{p} \frac{u_i^H y_f u_i}{\|u_i\|_2^2} \tag{10}$$

Where p is number of seismic waves. After rearranging the filtered signal subspace long vector y_{final} into multicomponent F-x seismic sections and performing an inverse Fourier transform, the filtered seismic data in time domain is obtained **(Figure6)**.

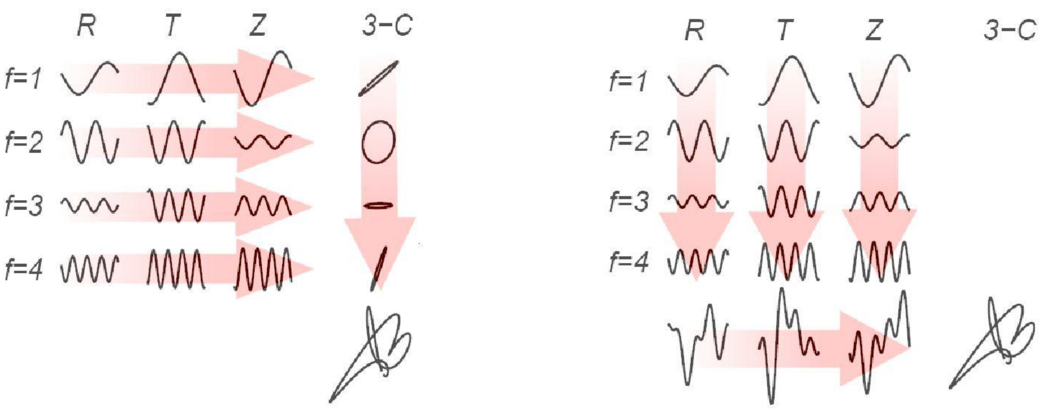

Figure1$^{(4)}$*: Alternatively, one could first combine the components at each frequency, to give a set of three-component Fourier ellipses. Adding over frequency then gives the whole trace as before. The properties of the ellipses provide us with a new set of Fourier spectra, described in subsequent figures (left figure). Fourier decomposition of a multicomponent synthetic trace. The trace can be assembled by summing over frequency to build up the R, T and Z components, then combining the components, as depicted with arrows (right figure).*

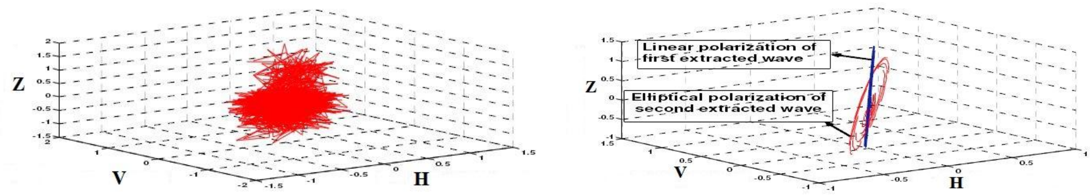

Figure2[1]: *Hodogram of initial multicomponent seismic wavefield data set (left figure). Hodogram of seismic wavefield after filtering (right figure).*

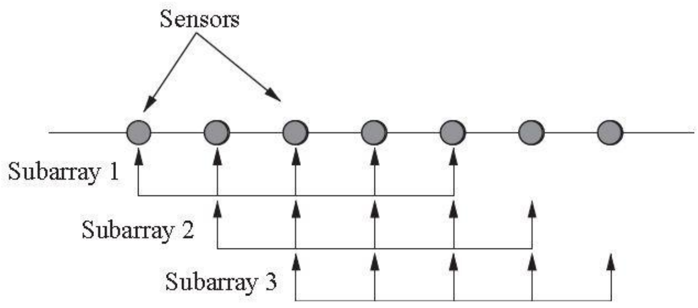

Figure3[3]: *Spatial smoothing: subarrays' design.*

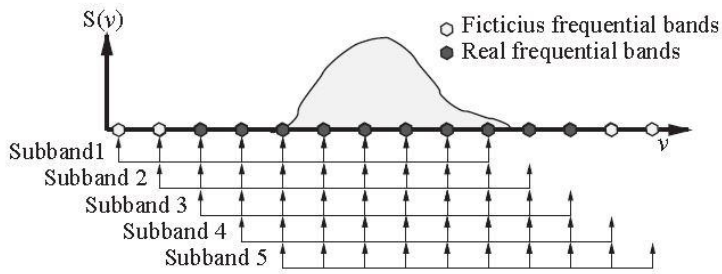

Figure4[3]:*Frequential smoothing: subbands' design.*

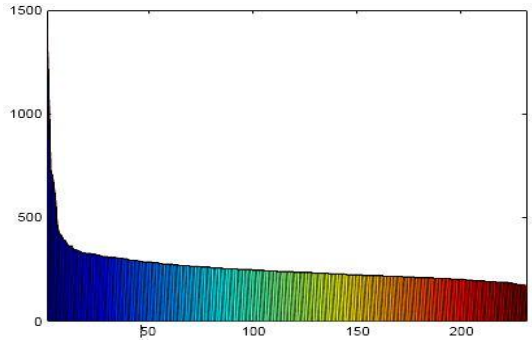

Figure5*: Eigenvalues of estimated reduced dimension covariance spectral matrix*

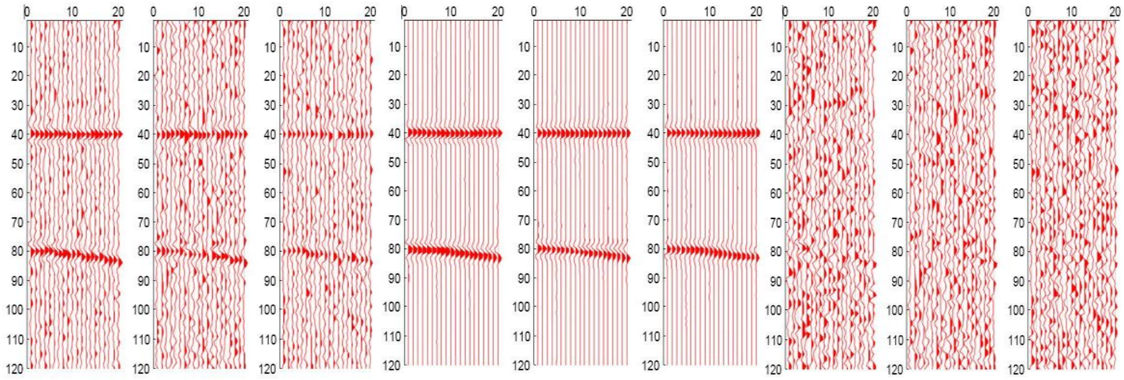

Figure6: The three right figures are input 3component synthetics seismic data set with added random noise. The three middle figures are same data set after applying the proposed method. The three right figures, which are random noises, are extracted from input 3component seismic data by removing filtered 3component seismic data (the three right figures).

Conclusions

This paper presents a method for filtering of multi-component seismic data. A significant advantage is reduction of spectral covariance matrix dimension. The eigenvalue decomposition of the reduced dimension covariance spectral matrix has been derived from the SVD. Comparing with other conventional algorithms, the proposed method has been extremely capable to reduce the computing time compared to the wideband spectral matrix algorithm as well as improving the accuracy in comparison with the SVD algorithm.

References

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