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Towards Real Earth Models - Computational Geophysics on Unstructured Tetrahedral Meshes?

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SUMMARY

Using unstructured tetrahedral meshes to specify 3D geophysical Earth models has a numer of advantages. Such meshes can conform exactly to the triangularly tessellated wireframe surfaces in the 3D Earth models used by geologists. This offers up the possibility of both geophysicists and geologists working with a single unified Earth model. Unstructured tetrahedral meshes are extremely flexible, and so can accurately mimic arbitrarily complicated subsurface structures and topography. Also, in the context of electromagnetic methods, unstructured tetrahedral meshes can be very finely discretized around sources and yet can transition to a coarse discretization in the extremities of the solution domain without, in principle, affecting the quality of the mesh. However, using unstructured tetrahedral meshes for geophysical Earth models has its challenges. The tessellated surfaces in wireframe geological models are often not immediately suitable for computational techniques as they can contain intersecting facets and facets with extreme aspect ratios. Generating tetrahedral meshes that are of sufficient quality from real wireframe geological models can therefore be difficult. This presentation will aim to discuss the pros and cons of using unstructured tetrahedral meshes for geophysical Earth models, keeping in mind the complexities of the real subsurface that we are ultimately trying to represent.

Introduction

When visualizing the subsurface in the context of mineral exploration, or when performing calculations such as estimating ore reserves, geologists use 3D Earth models that are made up of tessellated wireframe surfaces. For example, the left panel in Figure 1 shows the 3D wireframe geological model for the Voisey's Bay region of Labrador, Canada. Geologists also use wireframe models in the context of hydrocarbon exploration, in this case to model contacts between sedimentary units, including folding, and to model any faults cross-cutting the units.

In contrast, we geophysicists have so far used almost exclusively rectilinear meshes to parameterize our Earth models. This is simply because it is easier to derive and implement the discrete mathematics required by our numerical methods for rectilinear meshes rather than for other more general meshes. However, a geophysical model specified in terms of a rectilinear mesh can never be entirely consistent with a geological model specified in terms of wireframe surfaces. no matter how fine the discretization of the rectilinear mesh may be. Wouldn't it be better if our geophysical models were parameterized in a way that was consistent with – meshed seamlessly with – geological models? An Earth model could then be a single unified, integrated entity that served as both a geological model and a geophysical model.

Unstructured tetrahedral meshes: Advantages

Unstructured tetrahedral meshes enable volumes to be discretized in a way that is completely consistent with wireframe surfaces discretized in terms of triangles: each triangular facet of the wireframe surface becomes a facet of a tetrahedron in the volumetric discretization. One particular advantage of unstructured tetrahedral meshes in the geophysical context is that, in principle, the volumes in a wireframe geological model can simply be filled with tetrahedra and an appropriate numerical technique used to synthesize geophysical data. This would avoid any translation, and its inherent interpolation, extrapolation and averaging, between the geological model and the geophysical model. A second advantage is in constrained inversion. Because geological boundaries can be present in an unstructured tetrahedral geophysical model exactly as they are in the geological wireframe model, they can be used directly to constrain a geophysical inversion rather than a stepwise approximation of the boundary. We have been developing approaches for forward modelling and inversion of a range of geophysical data-types using unstructured tetrahedral Earth models (see, e.g., Lelièvre et al., 2012; Lelièvre and Farquharson, 2013).

Geophysical EM on unstructured tetrahedral meshes

A number of authors have presented methods for synthesizing geophysical EM data using unstructured tetrahedral Earth models, for example, Börner et al. (2008), Um et al. (2010), Schwarzbach et al. (2011), and Puzyrev et al. (2013). We have also been developing EM forward modelling methods. In particular, Ansari and Farquharson (2013) present a finite-element formulation that uses a total-field formulation, decomposition of the electric field into vector and scalar potentials, and edge element and nodal basis functions respectively to describe the approximate vector and scalar potentials. Jahandari

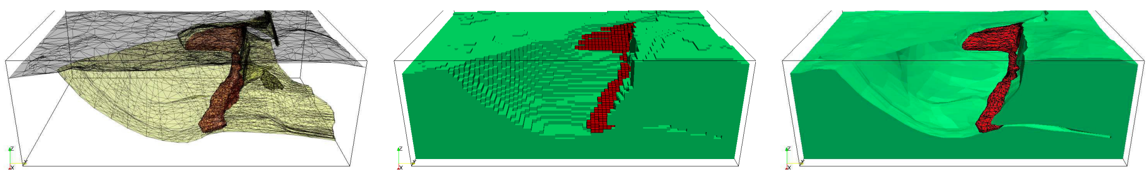

Figure 1 *Three simplified models of the main ore zones at Voisey's Bay, Labrador, Canada. Left panel: Geological model specified in terms of wireframe surfaces, with sulphide-troctolite contact in brown, troctolite-gneiss contact in yellow, and topography in grey. Middle panel: Geophysical model specified using a rectilinear mesh, with sulphides in red, gneiss in green, and the troctolite removed for illustration purposes. Right panel: Geophysical model specified in terms of an unstructured tetrahedral mesh.*

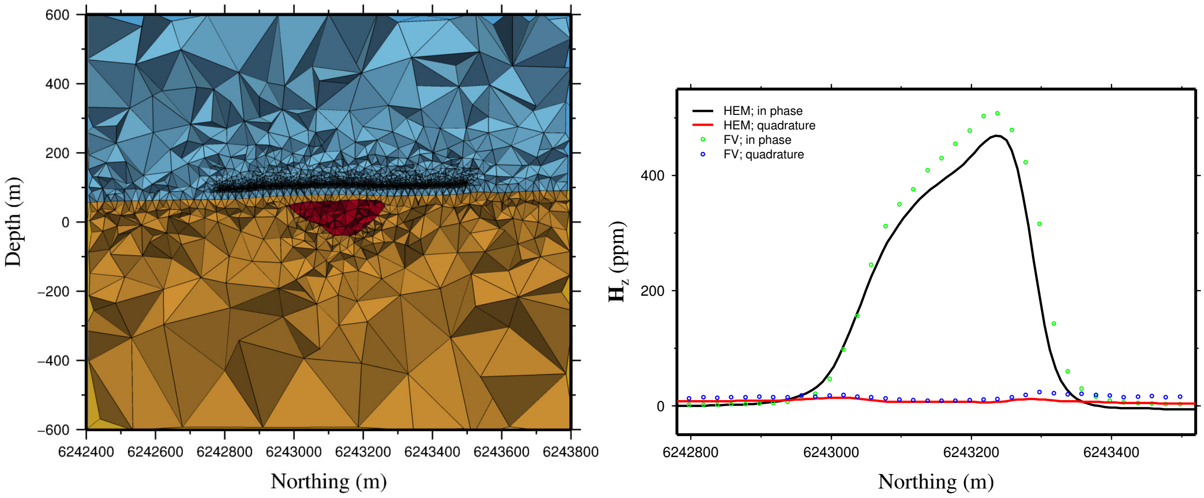

Figure 2 *Left panel: Vertical section through an unstructured tetrahedral mesh that was used to synthesize airborne EM data over the Ovoid ore-body at Voisey's Bay. Red corresponds to the Ovoid, brown to the background gneiss, and blue to air. Right panel: Real data (lines) and synthesized data (circles) along a profile over the Ovoid in the section shown in the left panel.*

and Farquharson (2013) present an extension to unstructured tetrahedral meshes of Yee's staggered-grid finite-difference approach, again using a total-field formulation. Figure 2 shows a comparison between data synthesized by the latter approach and real data for the Ovoid ore-body at Voisey's Bay. Figure 2 also illustrates a particular advantage of unstructured tetrahedral meshes for synthesizing EM data: these meshes allow for both very fine discretizations around sources and a transition to large cells towards the boundaries of the domain while nevertheless maintaining (in principle) the quality of the mesh.

Unstructured tetrahedral meshes: Challenges

There are, of course, challenges that come from using unstructured tetrahedral meshes to parameterize geophysical Earth models. One example is the generation of quality tetrahedral meshes from real geological wireframe models. Geological wireframe models are not designed with numerical computations in mind, and so often contain intersecting facets and facets with extreme aspect ratios. If these imperfections are not rectified, a tetrahedral mesh generated from the surfaces will inevitably be of poor quality. This can severely degrade the performance of many numerical methods.

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