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# New Advances for a Joint 3D Inversion of Multiple EM Methods

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## SUMMARY

Electromagnetic (EM) methods are routinely applied to image the subsurface from shallow to regional structures. EM methods differ in their sensitivities towards resistive and conductive structures as well as in their exploration depths. Joint 3D inversion of multiple EM data result in significantly better resolution of subsurface structures than the individual inversions. Proper weighting between different EM data is essential, however. We present a recently developed weighting algorithm to combine magnetotelluric (MT), controlled source EM (CSEM) and DC-geoelectric (DC) data. It is known that MT data are mostly sensible to regional conductive structures, whereas, CSEM and DC data are suitable to recover more shallow and resistive structures. Our new scheme is based on weighting individual components of the total data gradient after each model update. Norms of each data residual are used to assess how much weight individual components of the total gradient must have to achieve an equal contribution of all data in the inverse model. A numerically efficient way to search for appropriate weighting factors could be established by applying a bi-diagonalization to the sensitivity matrix. Thereby, the original inverse problem can be projected onto a smaller dimension in which the search of weighting factors is numerically cheap.

## Introduction

Electromagnetic (EM) methods vary in their resolution depths from few meters to hundreds of km. Magnetotelluric (MT) data is sensitive to conductive structures, reaching depths of the upper mantle, while controlled source EM (CSEM) and DC-geoelectrics (DC) have better resolution for resistive shallow structures. Inverting multiple EM data sets jointly can result in significantly better resolution of subsurface structures than the individual inversions. Because of different resolution capacity and sensitivity patterns of the EM methods, a proper weighting between the individual EM data sets is substantial. Earlier work in this regard was based on weighting the individual EM data sets by multiplying the error of each data set with a constant factor. Commer and Newman (2009), for instance, suggested up-weighting the data set with fewer data points by multiplying the assigned data errors with a scaling factor obtained from this ratio of available data. Weighting is applied once, as error floors and the number of data points remain unchanged in the course of the inversion. The assumption that the smaller data set must always get more weight is not generally valid. Moreover, applying the weighting only once in the course of a joint inversion, does not allow for a re-weighting if the data misfit of one data set converges to a desired level.

## Method and Example

In this study we propose new weighting schemes which are mainly based on analyzing the data gradients computed after each model update. In gradient based inversion algorithms (e.g. the Non Linear Conjugate Gradient, NLCG) an optimal penalty function is searched along the computed gradient using a line search algorithm (e.g., Egbert and Kelbert, 2012). In case of inverting two EM data sets jointly and analysis of the data gradients after each NLCG iteration, we observe that the norms of the data gradients differ typically by several orders of magnitude. The difference between the norms of the gradients reflects the resolution capacity and hence the sensitivity of each data set towards changes in model parameters. Without applying a proper weighting, structures required by the data set with a large gradient dominate the inverse model. Simply up-weighting the smaller gradient and down-weighting the larger gradient results in a more balanced distribution of structures required by both data sets.

Because of the non–quadratic form of the penalty function, a gradient based inversion algorithm requires a large number of iterations to reach the desired data misfit. In the Gauss-Newton approach in which the penalty function is quadratic, only few iterations are required. Solving the normal equations, which is necessary for the Gauss-Newton approach, is costly, however, in terms of memory usage and run times. Even if a Gauss-Newton variant in the data space is used (Siripunvaraporn and Egbert, 2000), memory requirements to store the full sensitivity matrix are huge. A possible solution to overcome memory problems is using the conjugate gradient (CG) method as a solver of the normal equations. Instead of saving the full sensitivity matrix, only matrix-vector multiplications are required to find a solution for the normal equations (e.g. Mackie and Madden, 1993). A disadvantage of the CG method is encountered when searching for an optimum value for the regularization parameter. In practice, for each value of the regularization parameter tested, the full set of normal equations has to be solved. Based on the Lanczos bi-diagonalization algorithms of Paige and Saunders (1982), Egbert (2012) showed that by saving the ortho-normal data and model search direction vectors computed in each CG iteration, the original dimension of the normal equation can be projected and stored in much smaller dimensions.

In our second approach of 3D joint inversion of multi-EM data sets, we extended the Hybrid CG-Occam inversion idea of Egbert (2012) to search additionally for an optimal value of weighting parameters. Our proposed weighting algorithm follows the same algorithm as described in Egbert (2012) up to the point where the model is updated. At that point we split the data gradients used to update the model into components related to each EM data type used in the joint inversion. Similar to the first proposed scheme, the gradient of each data set is up- or down-weighted based on the amplitude of the individual norms.

Using synthetic data sets we demonstrate the efficiency of the proposed weighting schemes and feasibility of 3D joint inversion of EM data. Figure 1 shows inversion results of applying our proposed gradient based weighting scheme together with results of inverting MT and CSEM data separately. Clearly, the joint inversion approach is best in recovering multi scale conductive and resistive structures.

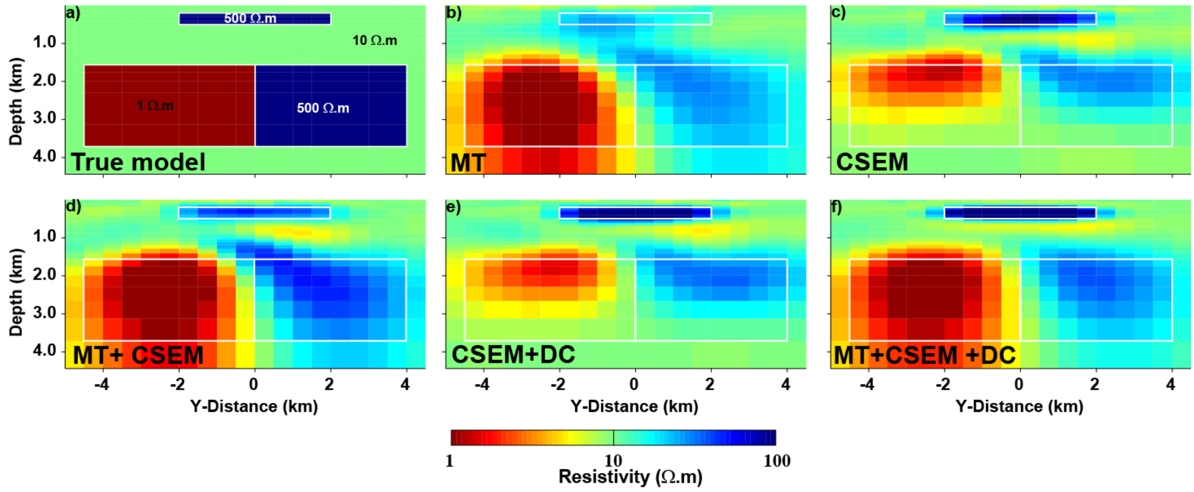


***Figure 1*** *Results obtained from inverting MT, CSEM and DC-Geoelectric data separately and jointly. The true 3-D model consists of a shallow and thin resistive block and two deeper conductive and resistive blocks. Shown here are 2D slices through the centre of the 3D models. For the joint inversion results we applied a new gradient based weighting scheme. The results show clearly the advantage of inverting multi-EM data jointly.*

## Conclusions

For joint 3D inversion of a range of EM data, we included three forward modelling operators (MT, CSEM and DC-Geoelectric) into the framework of the ModEM modelling/inversion package. The proposed gradient based weighting scheme is very efficient in balancing the contributions of the underlying MT, CSEM and DC data sets. Projection of the original problem into a Kyrlov- subspace using Lanczos bi-diagonalization is an efficient way to search for optimum regularization as well as weighting parameters.

## References

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