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Upscaling in Vertically Heterogeneous TTI Models

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SUMMARY

We define the low-frequency upscaling of vertically heterogeneous transversely isotropic layers with a tilted symmetry axis. In order to be used for velocity model building, the seismic wavelet is used as a weighting function. For a VTI medium, the low-frequency upscaling gives more accurate effective anisotropy parameters comparing with the ones obtained from Backus averaging. We also show that upscaling in TTI media results in much smaller effective tilt angle comparing with intrinsic one.

Introduction

To upscale well log data measured at sonic frequencies we assume that the medium is layered with layer thickness given by the logging step (commonly, 0.15m). Schoenberg and Muir (1981) extended the Backus (1962) solution for a medium composed of generally anisotropic layers. All such Backus-like methods are static (computed at zero frequency) and give no possibility of taking into account the frequency dependence of the upscaled medium. Stovas et al. (2013) proposed a method to extend the Backus upscaling method to low-frequencies and used the wavelet amplitude spectrum to weight averaging of the frequency-dependent effective parameters. In this paper, we extend this method to high-contrast media and TTI anisotropy. We show that the vertical averaging of elastic properties results is reducing the effective tilt angle. The larger vertical heterogeneity is the smaller effective tilt angle.

Low-frequency upscaling

The low-frequency (or zero-frequency) limit can be defined as the zero-frequency limit of the effective system matrix computed from a stack of the layers. This system matrix $\tilde{\mathbf{M}}$ is given by the logarithm of the product of non-commuting matrices from individual layers,

$$\tilde{\mathbf{M}}(\omega) = \frac{1}{i\omega H}\log\left(\exp\left(i\omega\Delta z\mathbf{M}_N\right)\ldots\exp\left(i\omega\Delta z\mathbf{M}_1\right)\right). \tag{1}$$

The logarithm in equation (1) is computed using an infinite series that can be illustrated for a periodically layered medium example, $N=2$. In this case, the matrix $\tilde{\mathbf{M}}$ is given by the Baker-Campbell-Hausdorff formula,

$$\begin{aligned}&\log\left(\exp(\mathbf{X})\exp(\mathbf{Y})\right) = \mathbf{X}+\mathbf{Y}+\frac{1}{2}[\mathbf{X},\mathbf{Y}]+\\ &\frac{1}{12}\left(\left[\mathbf{X},[\mathbf{X},\mathbf{Y}]\right]-\left[\mathbf{Y},[\mathbf{X},\mathbf{Y}]\right]\right)-\frac{1}{24}\left[\mathbf{Y},\left[\mathbf{X},[\mathbf{X},\mathbf{Y}]\right]\right]+\ldots\end{aligned}, \tag{2}$$

where $\mathbf{X}=\exp\left(i\omega\Delta z\mathbf{M}_2\right)$ and $\mathbf{Y}=\exp\left(i\omega\Delta z\mathbf{M}_1\right)$, and $[\mathbf{X},\mathbf{Y}]=\mathbf{XY}-\mathbf{YX}$ denotes the commuting operator. Note that the first term in series (2) represents the Backus average. Substituting series (2) into equation (1) results in the series in frequency (Roganov and Stovas, 2012),

$$\tilde{\mathbf{M}}(\omega) = \tilde{\mathbf{M}}_0 + i\omega\tilde{\mathbf{M}}_1 + (i\omega)^2\tilde{\mathbf{M}}_2 + \ldots, \tag{3}$$

with $\tilde{\mathbf{M}}_j$ being the matrix coefficients and $\tilde{\mathbf{M}}_0$ is the Backus system matrix.

Considering the effective matrix $\tilde{\mathbf{M}}$ as a system matrix we can find the eigenvalues corresponding to qP-, qSV- and qSH-wave modes. The frequency dependent effective medium is not a transversely isotropic medium with vertical symmetry axis (VTI) but possesses a vertical symmetry axis. This is due to the fact that the effective matrix $\tilde{\mathbf{M}}$ generally does not satisfy the equality

$$\mathbf{K}\tilde{\mathbf{M}}(\omega)\mathbf{K} = \tilde{\mathbf{M}}^T(\omega), \tag{4}$$

which is always valid for a homogeneous elastic or anelastic medium (Braga and Herrmann, 1992). Here, matrix $\mathbf{K}=\begin{pmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{I} & \mathbf{0}\end{pmatrix}$ and $\mathbf{I}$ is the unit 3x3 matrix. Nevertheless, this medium can be approximated as a VTI medium for quasi-vertical wave propagation. Expanding the eigenvalues of $\tilde{\mathbf{M}}(\omega)$ in a Taylor series with respect to horizontal slowness,

$$q^2(\omega) = q_0^2(\omega) + q_2^2(\omega)p^2 + q_4^2(\omega)p^4, \tag{5}$$

and fitting coefficients $q_0^2(\omega)$, $q_2^2(\omega)$ and $q_4^2(\omega)$ with corresponding coefficients for the vertical slowness squared in a VTI medium gives frequency dependent effective medium parameters. For velocity model building, these parameters are averaged using weights from the propagating wavelet spectrum.

Strong contrast TTI model

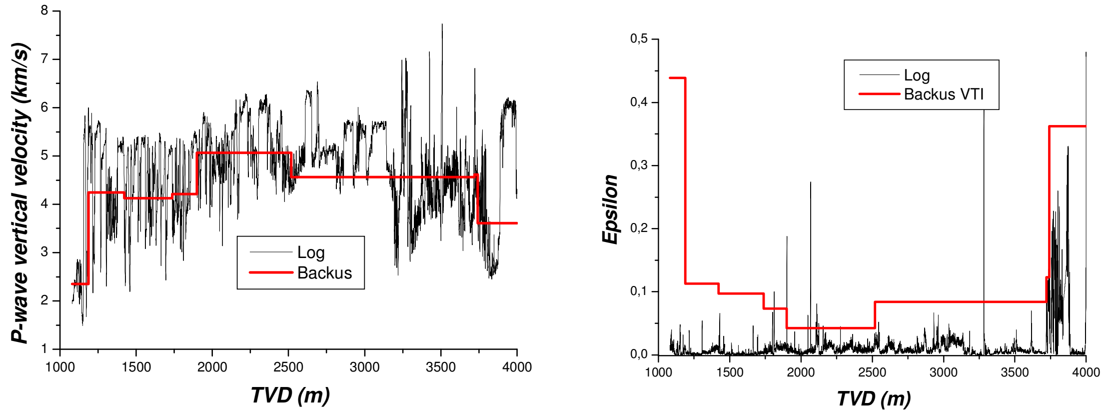
The well-log vertical P-wave velocity and anisotropy parameters ε and δ are plotted in Figure 1. The blocked values for corresponding Backus properties (computed under the VTI assumption) are shown in the same plots with red colours. Note that the Backus anisotropy properties have much stronger magnitudes than the intrinsic anisotropy parameters. This is an indication of the strong contrast in a layered medium. Since there is no information on intrinsic tilt values available, we simply assign a constant tilt value for each formation, $\theta = 30^o$. To compute low-frequency effective medium parameters, we use the approach proposed in Stovas et al. (2013) and obtain an effective slowness surface for each frequency harmonic, $q(p,\omega)$. Here we are faced with two major problems. The first problem is due to the fact that the TTI slowness surface is no longer symmetric about $p = 0$ and the tilt of the symmetry axis is unknown. The second problem is related to the discontinuity of the effective slowness surface in the presence of large contrasts in the medium. Theoretically, this effect was predicted and analyzed in Roganov and Stovas (2012). The discontinuities can appear both on the sides and on the top of the slowness surface. The structure of these discontinuities does not depend on intrinsic anisotropy, it is controlled by the contrast in isotropic elastic parameters. This follows from the theory of multipliers in layered media (Roganov and Stovas, 2012). If the slowness surface has a discontinuity at $p = 0$, the Taylor series expansion at that point is not defined. Another related problem is that the ends of the individual slowness sheets create caustics in the group domain because at these points, we have $dq/dp = \pm\infty$ (Roganov and Stovas, 2012). The effective vertical slowness versus horizontal slowness and frequency is shown in Figure 2, top. The effective slowness surface for individual frequencies and the corresponding vertical energy flux computed from equation containing the corresponding components of the displacement velocity - stress vector,

$$E = -\frac{1}{2}\mathrm{Re}\left(u_r \sigma^*_{rz} + u_z \sigma^*_{zz}\right), \tag{6}$$

are shown in Figure 2 top and bottom, respectively. The effective slowness surface computed from the strong-contrast medium becomes discontinuous for nonzero frequencies. The discontinuities can appear both at zero values (e.g. $f = 38Hz$) and non-zero values (e.g. $f = 20Hz$) of horizontal slowness. To overcome these problems in estimating the effective medium parameters, we propose fitting the low-frequency slowness surface with a slowness surface for a TTI medium using a least-squares method. The main idea behind this method is that we want to preserve the same symmetry class of the constituent layers in the effective medium (the true effective medium could possess much lower symmetry properties).

At frequencies near zero, the effective slowness surfaces are continuous and can be easily approximated by a TTI slowness surface. Figure 3 shows two effective TTI slowness surfaces computed for formations with different heterogeneity (weak contrast (left), and strong contrast (right)) computed at a frequency of 1Hz. We see that strong heterogeneity reduces the effective tilt. This is practically zero for the strong-contrast formation and still quite pronounced for the weak contrast one.

Figure 4 shows effective medium anisotropy parameters plotted versus frequency together with zero-frequency limits and low-frequency (averaged using a Ricker wavelet with central frequency of 15Hz and maximum frequency of 60Hz) for one TTI formation. The effective anisotropy parameters and tilt value vary with frequency, and the LF Ricker averaged properties are different from those obtained using the zero-frequency limit. There is also a significant reduction in effective tilt compared to the, constant, intrinsic tilt. The synthetic seismograms computed from the original well-log data and blocked models with properties computed using the zero-frequency limit and LF Ricker averaging are shown in Figure 5 (top). The corresponding velocity spectra are shown in Figure 5 (bottom). One can see that despite of many similarities between the blocked models, the one with LF Ricker averaging results in more accurate seismogram with better preserved moveout.



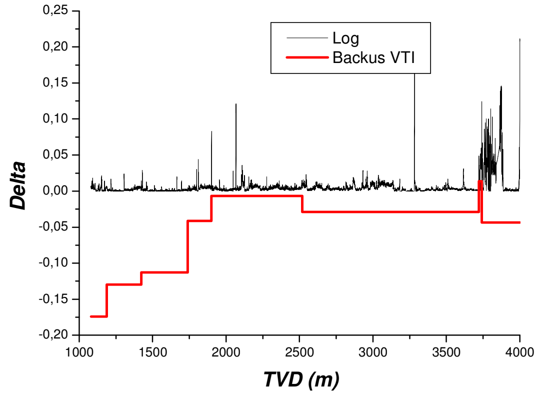


Figure 1 *The well-log data and the blocked model based on Backus averaging for a VTI medium.*

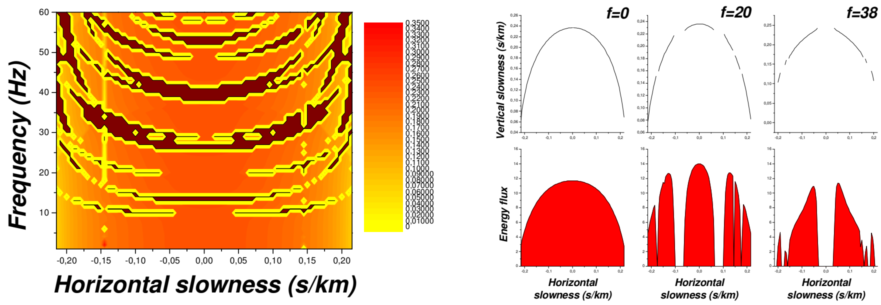


Figure 2 *The effective vertical slowness versus horizontal slowness and frequency (top) and the slices for individual frequencies together with corresponding vertical energy flux (bottom).*

Conclusions

We develop a new method for seismic upscaling of well-log data based on low-frequency wavelet averaging and tested this method for well-log data in the presence of strong contrasts in elastic properties for VTI and TTI media. The computed parameters for effective media are approximate in the sense that we want to have an effective medium with the same symmetry class as constitutive layers. We also show that a strong contrast in elastic parameters significantly reduces the tilt of the symmetry axis of the effective medium. The comparison between synthetic seismograms computed from the original well-log data and two blocked models with properties obtained from a zero-frequency limit and LF Ricker average shows that new upscaling method gives more accurate seismogram with better preserved moveout. This confirms the breakdown of Backus method in case of medium with strong contrast.

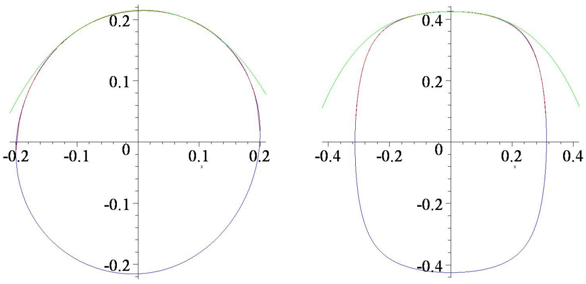


***Figure 3** The effective slowness surface computed for frequency of 1 Hz for weak-contrast (left) and strong-contrast (right) formations with constant intrinsic tilt of 30 degrees.*

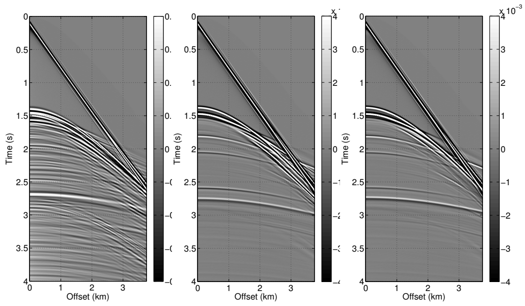


***Figure 5** Synthetic seismograms computed from orginal well-log data (left) and the blocked models with properties computed in zero-frequency limit (middle) and low-frequency Ricker averaging (right).*

Acknowledgements

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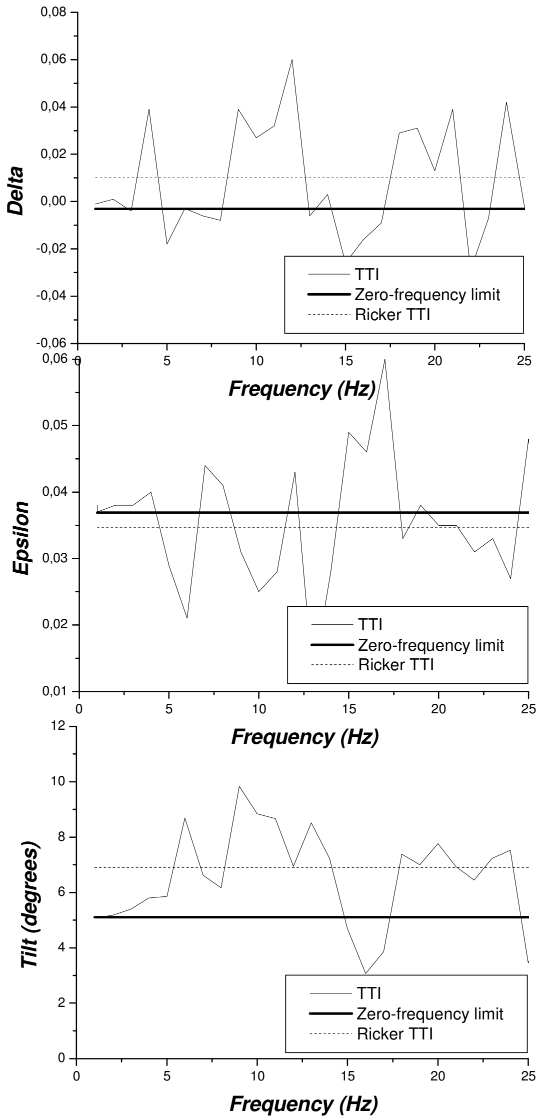

***Figure 4** Effective anisotropy parameters and effective tilt values with zero-frequency limit and low-frequency Ricker averaging.*

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